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SUGGESTED SOLUTION

FYJC 2020

SUBJECT- MATHEMATICS

Test Code - FYJ 6087

BRANCH - () (Date :)

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ANSWER : 1

(a) If n is even, $n = 2k$

$S_{2k} = (3-3) + (3-3) + (3-3) + \dots\dots\dots + (3-3) = 0.$

If n is odd, $n = 2k + 1$

$S_{2k+1} = S_{2k} + t_{2k+1} = 0 + 3 = 3.$

(02)

(b) Here a = 1, r = -3

AS $|r| \nless 1$

$\therefore$ Sum to infinity does not exist.

(02)

(c) let A_1, A_2 , A_3, A_4 be 4 arithmetic means between 2 and 22

$\therefore$ 2, A_1, A_2, A_3, A_4, 22 are in AP with

$a = 2$, $t_6 = 22$, n = 6.

$\therefore$ $22 = 2 + (6-1)d = 2 + 5d$

$20 = 5d$, $d = 4$

$A_1 = a + d = 2 + 4 = 6,$

$A_2 = a + 2d = 2 + 2 \times 4 = 2 + 8 = 10,$

$A_3 = a + 3d = 2 + 3 \times 4 = 2 + 12 = 14$

$A_4 = a + 4d = 2 + 4 \times 4 = 2 + 16 = 18.$

$\therefore$ the 4 arithmetic means between 2 and 22 are 6 , 10, 14, 18.

(02)

(d) r = $\frac{1}{3}$

Here a = $\frac{1}{3}$, r = $\frac{1}{3}$, $|r| < 1$

$\therefore$ Sum to infinity exists.

$$S = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \left[\frac{\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)}\right] = \frac{1}{2}$$

(02)

(e) $S_n = 5115 = 5\left(\frac{2^n-1}{2-1}\right) = 5\,(2^n - 1).$

$\therefore \frac{5115}{5} = 1023 = 2^n - 1$

$2^n = 1024 = 2^{10}$

$\therefore n = 10$

(02)

(f) $S_{10} = a\left(\frac{1-2^{10}}{1-2}\right) = a\,(1023),$

$\therefore a = 1$

(02)

ANSWER : 2

(a) Let the required numbers be $\frac{1}{H_1}$ and $\frac{1}{H_2}$

$\therefore \frac{2}{9}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{12}$ are in H.P.

$\frac{9}{2}$, H_1 , H_2 12 are in A.P.

$t_1 = a = \frac{9}{2}$, $t_4 = 12 = a + 3d = \frac{9}{2} + 3d.$

$3d = 12 - \frac{9}{2} = \frac{24-9}{2} = \frac{15}{2}$

$d = \frac{5}{2}$

$t_2 = H_1 = a + d = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7.$

$t_3 = H_2 = a + 2d = \frac{9}{2} + 2 \times \frac{5}{2} = \frac{19}{2}$...

For resulting sequence $\frac{1}{7}$ and $\frac{2}{19}$ are to be

Inserted between $\frac{2}{9}$ and $\frac{1}{12}$

(03)

(b) $t_n = \frac{5^{n-2}}{4^{n-3}}$

$t_{n+1} = \frac{5^{n-1}}{4^{n-2}}$

Consider $\frac{t_{n+1}}{t_n} = \frac{\frac{5^{n-1}}{4^{n-2}}}{\frac{5^{n-2}}{4^{n-3}}}$

$= \frac{5^{n-1}}{4^{n-2}} \times \frac{4^{n-3}}{5^{n-2}} = \frac{5}{4}$ = constant,

$\forall$ n $\in$ N.

The given sequence is a G.P. with $r = \frac{5}{4}$ and $t_1 = a = \frac{5^{1-2}}{4^{1-3}} = \frac{5^{-1}}{4^{-2}} = \frac{16}{5}$.

(03)

(c) (i) 0.66666…… = 0.6 + 0.06 + 0.006 + ………

$= \frac{6}{10} + \frac{6}{100} + \frac{6}{1000} + \ldots\ldots\ldots\ldots$,

the terms are in G.P. with a = 0.6, r= 0.1 < 1

$\therefore$ Sum to infinity exists and is given by

$\frac{a}{1-r} = \frac{0.6}{1-0.1} = \frac{0.6}{0.9} = \frac{6}{9} = \frac{2}{3}$

(ii) $0.\overline{46}$ = 0.46 + 0.0046 + 0.000046 + …….. the terms are in G.P. with a = 0.46, r = 0.01 < 1.

$\therefore$ Sum to infinity exists

$= \frac{a}{1-r} = \frac{0.46}{1-(0.01)} = \frac{0.46}{0.99} = \frac{46}{99}$

(03)

(d) Let the four numbers be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ (common ratio is r^2)

According to the first condition

$\frac{a}{r^3} \times \frac{a}{r} \times ar \times ar^3 = 64$

$\therefore a^4 = 64$

$\therefore\ a = 2\sqrt{2}$.

Now using second condition $\frac{a}{r} + ar$ = 6

$\frac{2\sqrt{2}}{r} + 2\sqrt{2}r$ = 6, dividing by 2 we get,

$\frac{\sqrt{2}}{r} + \sqrt{2}\,r$ = 3 now multiplying by r we get

$\sqrt{2} + \sqrt{2}r^2 - 3r = 0$

$\sqrt{2}\,r^2 - 3r + \sqrt{2}$ = 0,

$\sqrt{2}\,r^2 - 2r - r + \sqrt{2}$ = 0,

$\sqrt{2}\,r\left(r - \sqrt{2}\right) - 1\left(r - \sqrt{2}\right)$ = 0.

r = $\sqrt{2}$ or r = $\frac{1}{\sqrt{2}}$

If a = $2\sqrt{2}$, and r = $\sqrt{2}$ then 1, 2, 4, 8 are the four required numbers

If a = $2\sqrt{2}$, and r = $\frac{1}{\sqrt{2}}$ then 8, 4, 2, 1 are the four required numbers in G.P.

(03)

ANSWER : 3

(a) Let S_n

= 5 + 55 + 555 + 5555 + ……. upto n terms

= 5(1 + 11 + 111 + ………….. upto n terms)
= $\frac{5}{9}$ (9 + 99 + 999 + …………… upto n terms)

= $\frac{5}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + ………… to n brackets]

= $\frac{5}{9}$ [(10 + 100 + 1000 + ……. upto n terms)

– (1 + 1 + 1 + ……….. upto n terms)]

= $\frac{5}{9}$ [10 $\left(\frac{10^n - 1}{10 - 1}\right) - n$]

$= \frac{5}{9} [\frac{10}{9} (10^n - 1) - n]$

(04)

(b) $S_n = 5(4^n - 1)$, $S_{n-1} = 5(4^{n-1} - 1)$

We know that $t_n = S_n - S_{n-1}$

$= 5(4^n - 1) - 5(4^{n-1} - 1)$

$= 5(4^n) - 5 - 5(4^{n-1}) + 5$

$= 5(4^n - 4^{n-1})$

$= 5(4^n - 4^n \cdot 4^{-1})$

$= 5(4^n - (1 - \frac{1}{4})$

$= 5(4^n)\left(\frac{3}{4}\right)$

$\therefore\ t_{n+1} = 5(4^{n+1}) \times \frac{3}{4}$

Consider $\frac{t_{n+1}}{t_n} = \frac{5(4^{n+1})}{5(4^n)}$ = 4 = constant,

$\forall\ n \in N$.

$\therefore\ r = 4$

$\therefore$ the sequence is a G.P.

(04)

(c) Given A = G + 2 $\quad \therefore$ G = A – 2

Also $A = H + \frac{18}{5} \quad \therefore\ H = A - \frac{18}{5}$

We know that $G^2 = A H$

$(A - 2)^2 = A(A - \frac{18}{5})$

$A^2 - 4A + 2^2 = A^2 - \frac{18}{5} A$

$\frac{18}{5} A - 4A = -4$

$-2A = -4 \times 5$, $\therefore$ A = 10

Also G = A – 2 = 10 – 2 = 8

$\therefore$ A = $\frac{x+y}{2}$ = 10, x + y = 20, y = 20 – x .... (i)

Now G = $\sqrt{xy}$ = 8 $\therefore$ xy = 64

$\therefore$ x (20 – x) = 64

$20x - x^2 = 64$

$x^2 - 20x + 64 = 0$

(x – 16) (x – 4) = 0

x = 16 or x = 4

$\therefore$ If x = 16, then y = 4 $\therefore$ y = 20 – x

$\therefore$ If x = 4, then y = 16.

The required numbers are 4 and 16.

(04)

(d) Let the three numbers be $\frac{a}{r}$, *a, ar*.

According to first condition their sum is 42.

$$\therefore \frac{a}{r} + a + ar = 42$$

$$a\left(\frac{1}{r} + 1 + r\right) = 42$$

$$\therefore \frac{1}{r} + 1 + r = \frac{42}{a}$$

$$\frac{1}{r} + r = \frac{42}{a} - 1 \text{.............. (1)}$$

From the second condition their product is 1728

$$\frac{a}{r} \cdot a \cdot ar = 1728$$

$$\therefore a^3 = 1728 = (12)^3$$

$\therefore$ a = 12.

Substitute a = 12 in equation (1), we get

$$\frac{1}{r} + r = \frac{42}{12} - 1$$

$$\frac{1}{r} + r = \frac{42 - 12}{12}$$

$$\frac{1}{r} + r = \frac{30}{12}$$

$$\frac{1 + r^2}{r} = \frac{5}{2}$$

$\therefore$ $2 + 2r^2 = 5r$

$\therefore$ $2r^2 - 5r + 2 = 0$

$\therefore$ (2r – 1) (r – 2) = 0

$\therefore$ 2r = 1 or r = 2

$\therefore$ r = $\frac{1}{2}$ or r = 2

Now if a = 12, and r = $\frac{1}{2}$ then the required numbers are 24, 12, 6.

(04)